2026 specimen: worked solutions

Paper 1: 90 marks | Paper 2: 90 marks

This is an indicative teaching mark scheme. Award equivalent valid methods and follow-through where appropriate. Bracketed marks show a reasonable division of credit; they are not an official marking policy. Retain unrounded values during calculations.

For graph answers, use a sensible tolerance based on the printed scale. Construction marks require visible, valid arcs. Statistical standard deviation throughout is the population standard deviation.

Paper 1

Question 1 · 3 marks

(a) $756 = 2^2 \times 3^3 \times 7$. One mark for a valid factorisation method; one for the complete prime-power form.

(b) $k = 2 \times 7^2 = 98$. The exponents in $756k$ must all be multiples of 3. [1]

Question 2 · 3 marks

(a) Upper bounds: 6.45 cm and 3.85 cm. [1] Area upper bound $= 6.45 \times 3.85 = 24.8325$ cm². [1]

(b) 7.28×10^{-5}. [1]

Question 3 · 3 marks

(a) $y = k/\sqrt{x}$, so $k = 12\sqrt{9} = 36$. [1] $y = 36/4 = 9$. [1]

(b) y is halved, since $\sqrt{(4x)} = 2\sqrt{x}$. [1]

Question 4 · 3 marks

(a) Original price $\times 1.08 \times 0.85 = 275.40$. [1] Original price = \$300. [1]

(b) $1.08 \times 0.85 = 0.918$, so the overall change is an 8.2% decrease. [1]

Question 5 · 3 marks

(a) $2x^2 + 10x - 3x - 15$ [1] $= 2x^2 + 7x - 15$. [1]

(b) $x = (5t + 4)/3$. [1]

Question 6 · 3 marks

(a) $-7 < -3x \leq 12$. Dividing by -3 reverses both inequalities. [1] $-4 \leq x < 7/3$. [1]

(b) Seven: $-4, -3, -2, -1, 0, 1, 2$. [1]

Question 7 · 3 marks

(a) {2, 4, 6, 12}. [1]

(b) {5, 7, 9, 11}. [1]

(c) 2, corresponding to {1, 3}. [1]

Question 8 · 4 marks

(a) MC = ((18×14 + 12×9), (25×14 + 16×9)) = ((360), (494)). One mark for each correct entry.

(b) Saturday’s total revenue in dollars. [1]

(c) 25 × (14 × 1.15) + 16 × 9 = \$546.50. [1]

Question 9 · 3 marks

(a) 34 − 18 = 16 minutes. [1]

(b) Group B has the lower median (25 rather than 26 minutes), so its typical time is slightly shorter. [1]

(c) Group B is more consistent: IQR = 30 − 22 = 8 minutes, compared with 16 minutes for Group A. [1]

Question 10 · 3 marks

(a) 90°, angle in a semicircle. [1]

(b) 180° − 90° − 32° = 58°, angles in a triangle. [1]

(c) 32°, alternate segment theorem (or radius perpendicular to tangent). [1]

Question 11 · 3 marks

(a) Linear scale factor = 3/2 = 1.5; area factor = 1.5^2. [1] Larger area = 96 × 2.25 = 216 cm^2. [1]

(b) Smaller volume = 145.8 × $(2/3)^3$ = 43.2 cm^3. [1]

Question 12 · 3 marks

(a) (−1 − 5)/(4 + 2) = −1. [1]

(b) Midpoint is (1, 2); perpendicular gradient is 1. [1] y − 2 = x − 1, so y = x + 1. [1]

Question 13 · 4 marks

(a) $(2x - 5)(2x + 5)$. [1]

(b) Numerator $= (2x + 1)(x - 3)$, denominator $= (2x - 1)(2x + 1)$. [1] Simplified expression $= (x - 3)/(2x - 1)$. [1]

(c) $x = 1/2$ and $x = -1/2$. Both exclusions remain necessary. [1]

Question 14 · 3 marks

(a) $x^2 - 6x + 9 + 2$ [1] $= (x - 3)^2 + 2$. [1]

(b) (3, 2). [1]

Question 15 · 3 marks

(a) Final amount $= 2500(1.032)^4$. [1] Interest $= 2500(1.032)^4 - 2500 = \$335.69$. [1]

(b) Fourth-year interest is calculated on a larger balance that includes interest from earlier years. [1]

Question 16 · 3 marks

(a) $18/12 = 1.5$ m/s^2. [1]

(b) Distance = area under graph $= \frac{1}{2}\times12\times18 + 20\times18 + \frac{1}{2}\times8\times18$. [1] $= 540$ m. [1]

Question 17 · 3 marks

(a) Differences are 6, 8, 10, then 12, so the fifth term is 41. [1]

(b) Second difference 2 gives a leading term n^2. Subtracting n^2 leaves 4, 7, 10, 13 $= 3n + 1$. [1] $T_n = n^2 + 3n + 1$. [1]

Question 18 · 3 marks

(a) Exterior angle $= 180° - 150° = 30°$. [1] Number of sides $= 360/30 = 12$. [1]

(b) Remaining angle $= 360° - 150° - 90° = 120°$. [1]

Question 19 · 4 marks

(a) Angle AOB $= 70°$. [1] $AB^2 = 8^2 + 11^2 - 2\times8\times11\cos70°$. [1] AB = 11.2 km (11.1716…). [1]

(b) 250°. [1]

Question 20 · 4 marks

(a) Area $= \frac{1}{2}r^2\theta = \frac{1}{2}\times12^2\times1.4$ [1] $= 100.8$ cm^2 (101 cm^2 to 3 s.f.). [1]

(b) Arc length $= 12\times1.4 = 16.8$ cm. [1] Perimeter $= 24 + 16.8 = 40.8$ cm. [1]

Question 21 · 4 marks

(a) AB = (6, 8), so magnitude = $\sqrt{(6^2 + 8^2)}$ [1] = 10. [1]

(b) AP = ¼AB = (1.5, 2). [1] P = (3.5, 1). [1]

Question 22 · 4 marks

(a) $(5\times4 + 3\times2 + 2\times1)/(10\times9)$. [1] = 28/90 = 14/45. [1]

(b) (2/10)(8/9) + (8/10)(2/9). [1] = 32/90 = 16/45. [1]

Question 23 · 4 marks

(a) $\Sigma fx = 35$, $\Sigma fx^2 = 81$, $\Sigma f = 20$. Mean = 1.75. [1] Population standard deviation = $\sqrt{(81/20 - 1.75^2)}$ = 0.994. [1]

(b) Mean = 3.75. [1] Standard deviation remains 0.994 because adding a constant leaves every deviation from the mean unchanged. [1]

Question 24 · 3 marks

(a) Equal-radius arcs centred at A and B, of radius greater than 4 cm, intersect above and below AB. [1] Join their intersections. [1]

(b) P is the upper intersection of the perpendicular bisector with the circle centred at A, radius 5 cm. It lies 3 cm above the midpoint. [1]

Question 25 · 5 marks

Multiply by $(x - 1)(x + 2)$: $5(x + 2) - 2(x - 1) = (x - 1)(x + 2)$. [1]

Expand to $3x + 12 = x^2 + x - 2$. [1] Hence $x^2 - 2x - 14 = 0$. [1]

$x = (2 \pm \sqrt{60})/2 = 1 \pm \sqrt{15}$. [1] x = 4.87 or −2.87. Neither is an excluded value (1 or −2). [1]

Question 26 · 6 marks

(a) Correct straight line through points such as (0, 2), (−2, 0) and (3, 5). One mark for two accurate points, one for the line.

(b) $x \approx -2.00, -0.41, 2.41$. Accept sensible graph readings, normally within 0.1. [2]

(c) $-2 \le x \le -0.41$ or $2.41 \le x \le 3$, with endpoints included. [2] These are the intervals where the curve lies on or above the line.

Paper 2

Question 1 · 10 marks

(a) Common denominator $(x - 2)(x + 1)$. [1] Numerator $x(x + 1) - 3(x - 2)$ [1] $= x^2 - 2x + 6$. Answer $(x^2 - 2x + 6)/((x - 2)(x + 1))$. [1]

(b) $2as = v^2 - u^2$ [1], so $s = (v^2 - u^2)/(2a)$, assuming $a \neq 0$. [1]

(c) $y = 4x - 1$. Substitution gives $2x + 3(4x - 1) = 11$. [1] $14x = 14$, $x = 1$. [1] $y = 3$. [1]

(d) Use $2n - 1$ and $2n + 1$. Their product plus 1 is $4n^2 - 1 + 1 = 4n^2$. [1] This is divisible by 4 for every positive integer n. [1]

Question 2 · 10 marks

(a) Midpoints are 5, 15, 25, 40. $\Sigma ft = 1470$ and $\Sigma ft^2 = 44650$. Mean $= 1470/60 = 24.5$ minutes. [1] SD $= \sqrt{(44650/60 - 24.5^2)} = 12.0$ minutes (11.9965…). [1]

(b) $20 < t \leq 30$. Cumulative frequencies are 8, 22, 42, 60. [1]

(c) Individual times within the median class are unknown. [1]

(d) $(38/60)(37/59)$ [1] $= 703/1770 \approx 0.397$. [1]

(e) There are 22 participants at or below 20 minutes and 38 above. For one order: $(22/60)(21/59)(38/58)$. [1] There are three possible positions for the participant above 20 minutes. [1] Probability = 3 times this product $= 4389/17110 \approx 0.257$. [1]

(f) Densities are $20/10 = 2$ and $18/20 = 0.9$. Required height $= 8 \times 0.9/2 = 3.6$ cm. [1]

Question 3 · 9 marks

(a) Cylinder volume $= \pi \times 3^2 \times 8 = 72\pi$. [1] Hemisphere volume $= (2/3)\pi \times 3^3 = 18\pi$. [1] Capacity $= 90\pi = 283$ cm³ (282.743…). [1]

(b) Water above the hemisphere has volume $60\pi - 18\pi = 42\pi$. [1] Cylindrical water height $= 42\pi/(9\pi) = 14/3$ cm. [1] Depth from lowest point $= 3 + 14/3 = 23/3$ cm $= 7.67$ cm. [1]

(c) Cylinder curved area $= 2\pi \times 3 \times 8 = 48\pi$. [1] Hemisphere curved area $= 2\pi \times 3^2 = 18\pi$. Total $= 66\pi$. [1] Cost $= 0.08 \times 66\pi = \$16.59$. [1]

Question 4 · 10 marks

(a) $BC^2 = 120^2 + 85^2 - 2 \times 120 \times 85 \cos 58°$. [2] BC = 104 m (103.9935…). [1]

(b) Area $= \frac{1}{2} \times 120 \times 85 \times \sin 58°$ [1] $= 4325.045\ldots \approx 4330$ m². [1]

(c) $\frac{1}{2} \times BC \times AP$ = area. [1] AP $= 2 \times 4325.045\ldots/103.9935\ldots = 83.2$ m (83.1792…). [1]

(d) $\tan \theta = 18/AP$ [1], so $\theta = 12.2°$. [1] AP is the shortest horizontal distance from A to BC, so it gives the greatest elevation angle for a fixed mast height. [1]

Question 5 · 11 marks

(a) $p = 1.5 + 6/1.5 = 5.5$. [1]

(b) Accurate plotting of all eight points [2] and a smooth curve with an appropriate minimum between $x = 2$ and $x = 3$ [1]. See the solution graph.

(c) $x \approx 1.27$ and 4.73. Accept graph readings within about 0.1. [2]

(d) A suitable tangent at (2, 5), with a large gradient triangle. [1] Gradient ≈ -0.5; allow roughly -0.7 to -0.3 for a careful drawing. [1]

(e) Correct line through (0.5, 0), (3, 5) and (5, 9). [1]

(f) $x + 6/x = 2x - 1$ gives $x^2 + 6 = 2x^2 - x$, hence $x^2 - x - 6 = 0$. [1] Roots are 3 and -2; -2 is outside the plotted domain. [1]

Question 6 · 10 marks

(a) $\overrightarrow{OP} = (2/5)\mathbf{a}$, $\overrightarrow{OQ} = \mathbf{a} + (2/3)(\mathbf{b} - \mathbf{a}) = (1/3)\mathbf{a} + (2/3)\mathbf{b}$. [1] $\overrightarrow{PQ} = -(1/15)\mathbf{a} + (2/3)\mathbf{b}$. [1]

(b) $\overrightarrow{OR} = \overrightarrow{OP} + h\overrightarrow{PQ} = (2/5 - h/15)\mathbf{a} + (2h/3)\mathbf{b}$. [1] Since a and b are not parallel, $2/5 - h/15 = 0$, giving $h = 6$. [1] $k = 2h/3 = 4$. [1]

(c) Area OPQ / area OAQ = OP/OA = 2/5 (same altitude from Q). Area OAQ / area OAB = AQ/AB = 2/3. [1] Product = 4/15, so ratio 4 : 15. [1]

(d) P = (4, 2); R = 4B = (−12, 24). [1] Gradient = $(24 - 2)/(-12 - 4) = -11/8$. [1] $y - 2 = -(11/8)(x - 4)$, or $11x + 8y = 60$. [1]

Question 7 · 10 marks

(a) $48/x$ hours. [1]

(b) $48/x - 48/(x + 4) = 1/2$. [1] Multiplying by $2x(x + 4)$ gives $96(x + 4) - 96x = x(x + 4)$. [1] Therefore $x^2 + 4x - 384 = 0$. [1]

(c) $x = (-4 \pm \sqrt{1552})/2 = -2 \pm 2\sqrt{97}$. [1] Roots ≈ 17.6977 and -21.6977. [1] Speed must be positive, so outward speed = 17.7 km/h. [1]

(d) Return time = $48/(17.6977\ldots + 4) = 2.212214\ldots$ hours. [1] = 2 hours 13 minutes. [1]

(e) The cyclist spends longer at the lower speed, so the time-weighted mean speed is below the unweighted arithmetic mean. [1]

Question 8 · 10 marks

(a) Sum = 5x, so x = 215/5 = 43. [1]

(b) Horizontal product = $(x - 1)(x + 1) = x^2 - 1$. [1] Vertical product = $(x - 7)(x + 7) = x^2 - 49$. [1] Difference = 48, independent of x. [1]

(c) $T_{n+1} = (n + 1)^2 + 4(n + 1) + 2 = n^2 + 6n + 7$. [1] Subtract $n^2 + 4n + 2$ to get $2n + 5$. [1]

(d) $2n + 5 = 51$ gives n = 23. [1] The 23rd and 24th terms. [1]

(e) $n^2 + 4n + 2 = 2026$ gives $(n + 2)^2 = 2028$. [1] Since $45^2 = 2025$ and $46^2 = 2116$, 2028 is not a perfect square; there is no positive integer n. Therefore 2026 is not a term. [1]

Question 9 · 10 marks

(a) Mean rainfall = 1000/6 mm per month. [1] Annual collection = 180 × (1000/6 × 12 / 1000) × 0.75 × 1000 = 270000 litres. [1]

(b) Annual need = 220×365 = 80300 litres. [1] Rainfall timing and limited storage mean water can overflow in wet periods and run out in a dry spell. Annual totals alone do not establish supply at every time. [1]

(c) A 21-day dry period needs 21×220 = 4620 litres. Tank A’s 5000-litre usable capacity is sufficient. [1] A’s 1.8 m diameter fits the 2.0 m square space; B’s 2.1 m diameter does not. [1]

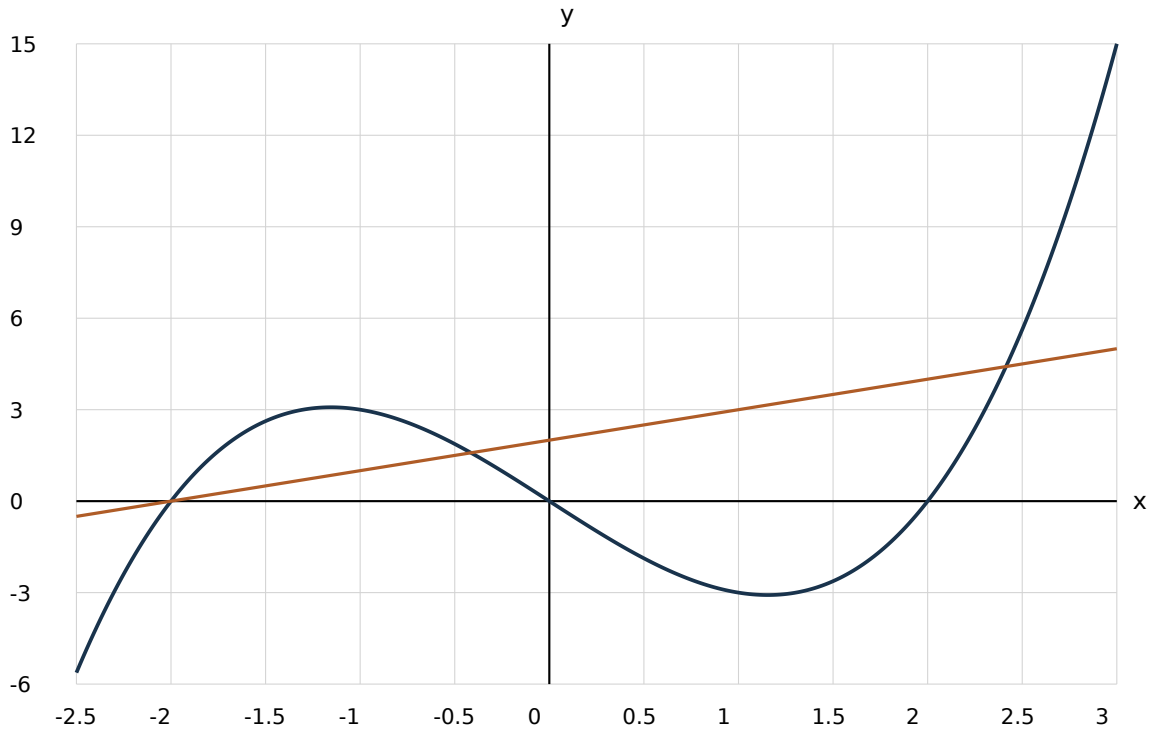
Tank A’s total cost = 1250 + 480 = $1730. [1] Annual mains-water saving = 0.70×80300/1000×2.85 = $160.1985. [1]

Simple payback = 1730/160.1985 = 10.8 years, below 12 years. [1] Recommend A, assuming no significant water is lost through leakage or evaporation during storage. Alternatively, assume stored rainwater remains suitable for watering the plants. State a relevant additional assumption, rather than merely repeating the supplied full-tank or 70% assumptions. [1]

Equivalent valid reasoning earns credit. Do not use 270000 litres as the bill-saving volume: savings are limited by the garden’s usage and the stated 70% replacement.

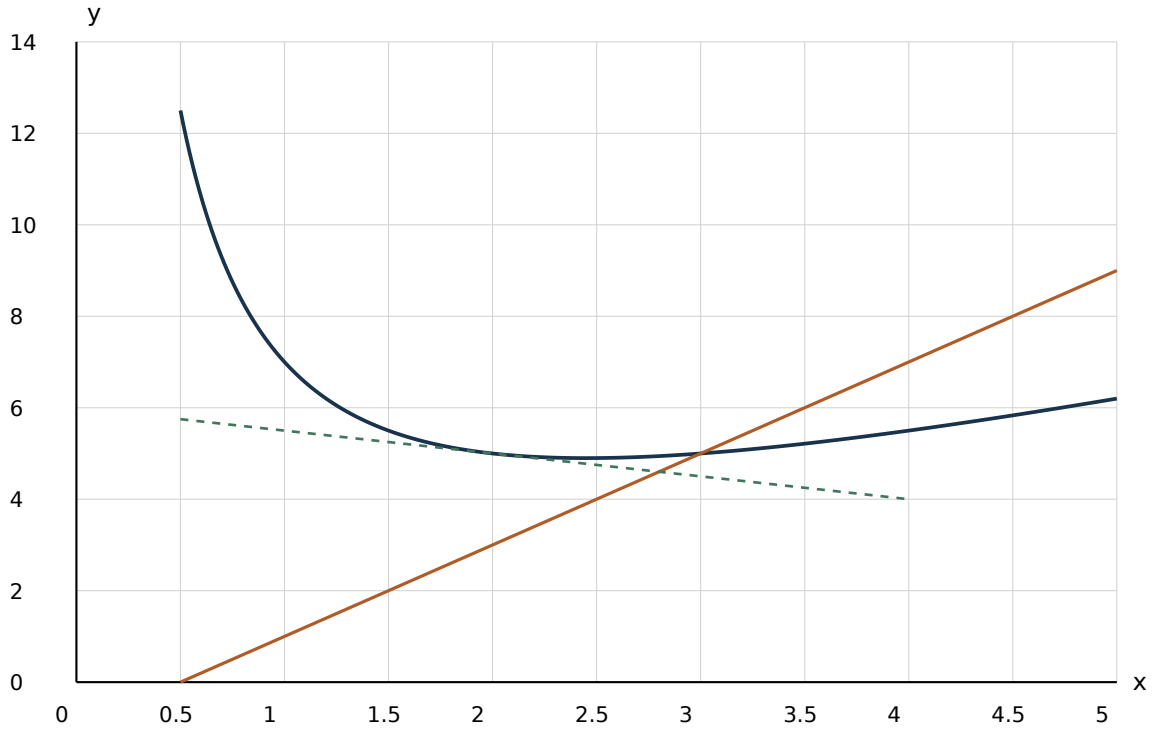
Graph answers

Paper 1, Question 26



Blue: $y = x^3 - 4x$. Brown: $y = x + 2$. Read the intervals where blue is on or above brown.

Paper 2, Question 5



Blue: $y = x + 6/x$. Brown: $y = 2x - 1$. Green dashed: tangent at (2, 5).